Volumes of Solids with Known Cross-Sections

Sometimes the bounded area is the base (or bottom) of a solid object. Think of a model building sitting on your graph paper. The base is the bounded region. The building grows up from the paper.

The problem will define the bounded area.
It will also tell you the shape of the cross sections (the things sticking up out of your paper).
It will tell you whether the shapes are perpendicular to the x-axis or y-axis. Integrate accordingly.
Finally the shapes of the cross sections will always be shapes that have an easy-to-find area formula.

To find the volume of a solid with known cross sections that are perpendicular to the x-axis, use this formula:

To find the volume of a solid with known cross sections that are perpendicular to the y-axis, use this formula:

Important area formulas:
Area of a square:

Area of a triangle:

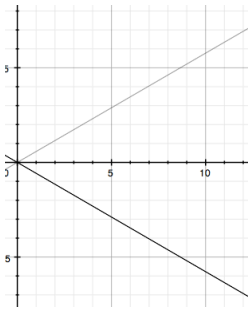
Area of an equilateral triangle:

Height of an equilateral triangle:

Area of an isosceles right triangle if you know one of the legs:

Area of an isosceles right triangle if you only know the hypotenuse:

1. Find the volume of the solid whose base is bounded by the lines
$y=\frac{1}{\sqrt{3}}x$ $y=\frac{-1}{\sqrt{3}}x$ and x= 5✓3 and whose cross-sections are squares perpendicular to the x-axis.

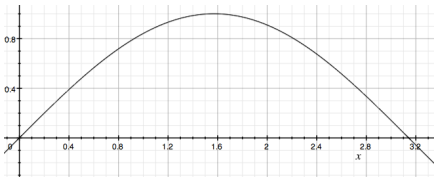


- shade the area indicated (5✓3 = 8.66)
- What is the area formula for a square?
- What equation gives the bottom side of the square on this graph?

- What are the bounds of integration?

Write the integral and use your calculator to evaluate the volume:

2. Find the volume of the solid whose base is bounded by y=sinx the x−axis, x=0, x= π and whose cross sections are equilateral triangles perpendicular to the x−axis.



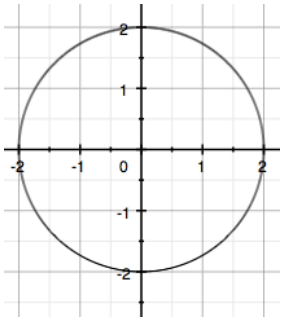
- shade the area indicated
- What is the area formula for an equilateral?

- What equation gives the bottom side of the equilateral on this graph?

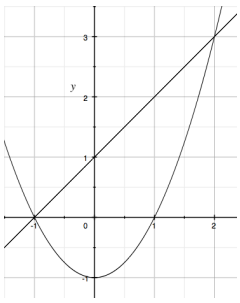
- What are the bounds of integration?

Write the integral and use your calculator to evaluate the volume:

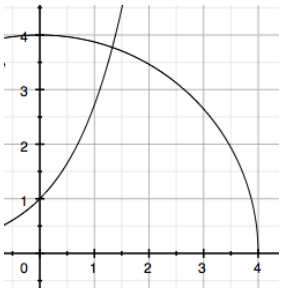
3. Find the volume of the solid whose base is a circle with radius 2. The cross sections are right isosceles triangles with the hypotenuse perpendicular to the x−axis.



4. Find the volume of a solid whose base is bounded by $y=x+1$ and $y=x^2-1$ and whose cross sections are rectangles of height = 2 perpendicular to the x-axis.



5. Find the volume of a solid whose base is bounded by $y^2+x^2=16$ and $y=e^x$ and $y=1$ and whose cross sections are squares perpendicular to the y-axis.



•Rewrite each equation as x=

• Find the bounds of integration. For the upper bound, you will have to use your calculator's graphing function to estimate. Graph the original equations; hit calc (2nd trace); and select #5 intersect. You have to identify the curves and get close to the intersection.

•Write the equation that gives you the length of the side of the square. Remember that the square is perpendicular to y-axis. You'll have to figure this one out.

• Write the integral and evaluate with a calculator.